Section 2 Annubar Primary Element Flow Calculations

ANNUBAR PRIMARY ELEMENT FLOW EQUATIONS

The Annubar primary element flow equations are all derived from the hydraulic equations which are shown on page 1-11. For a detailed example of a derivation of an Annubar primary element equation, see the Rosemount 485 Annubar Flow Test Data Book (document number 00821-0100-4809).

Equation 2-1. : Volume rate of flow - Liquids (Actual Conditions)

$$Q_a = C' \cdot \sqrt{h_w} \qquad \text{OR} \qquad h_w = \left(\frac{Q_a}{C'}\right)^2$$

where:

$$C' = F_{na} \cdot K \cdot D^2 \cdot F_{aa} \cdot \sqrt{\frac{1}{G_f}}$$

NOTE:
For description of standard volumetric flow equations, see page 1-12.

Equation 2-2. : Mass rate of flow - Liquids

$$W = C' \cdot \sqrt{h_w} \qquad \text{OR} \qquad h_w = \left(\frac{W}{C'}\right)^2$$

where:

$$C' = F_{na} \cdot K \cdot D^2 \cdot F_{aa} \cdot \sqrt{\rho_f}$$

Equation 2-3. : Mass rate of flow - Gas and Steam

$$W = C' \cdot \sqrt{h_w} \qquad \text{OR} \qquad h_w = \left(\frac{W}{C'}\right)^2$$

where:

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{aa} \cdot \sqrt{\rho_f}$$

Equation 2-4. : Volume rate of flow - Gas (Standard Conditions)

$$Q_s = C' \cdot \sqrt{h_w \cdot P_f} \qquad \text{OR} \qquad h_w = \frac{1}{P_f} \cdot \left(\frac{Q_s}{C'}\right)^2$$

where:

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{pb} \cdot F_{tb} \cdot F_{tf} \cdot F_g \cdot F_{pv} \cdot F_{aa}$$

Equation 2-5. : Volume rate of flow - Gas (Actual Conditions)

$$Q_a = C' \cdot \sqrt{h_w} \qquad \text{OR} \qquad h_w = \left(\frac{Q_a}{C'}\right)^2$$

where:

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y \cdot F_{aa} \cdot \sqrt{\frac{1}{\rho_f}}$$

For a detailed description of each term in the above equations, see “Nomenclature” on page 2-6. Please note that each of the above equations has a C' constant. It is not intended that the C' constant of one equation is equal to the C' constant of another equation. The numerical value of any C' constant is the product of the appropriate factors for that equation only.

The following tabulations of the flow equations will serve as handy work pads. Also, the table numbers where the necessary information can be found are given in the headings of these tabulations. Several completed examples of flow calculations are given beginning on page 2-11.

NOTE
The 485 Annubar primary element needs no correction for the Reynolds Number.

Table 2-1. Equation for Liquid – Volume Rate of Flow

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Thermal Expansion Factor (Table A-11)	Flowing Specific Gravity	Differential Pressure
	Annubar Flow Constant C^l					
Q_a =	F_{na} ·	K ·	D^2 ·	F_{aa} ·	$\sqrt{\frac{1}{G_f}}$ ·	$\sqrt{h_w}$
GPM	5.6664		$(in)^2$			inch H_2O at 68 °F
GPH	339.99		$(in)^2$			inch H_2O at 68 °F
GPD	8159.7		$(in)^2$			inch H_2O at 68 °F
BPH (42 gal)	8.0949		$(in)^2$			inch H_2O at 68 °F
BPD (42 gal)	194.28		$(in)^2$			inch H_2O at 68 °F
ft^3/min	0.75749		$(in)^2$			inch H_2O at 68 °F
CFH	45.4494		$(in)^2$			inch H_2O at 68 °F
CFM	0.7575		$(in)^2$			inch H_2O at 68 °F
Imp. GPM	4.7183		$(in)^2$			inch H_2O at 68 °F
LPH	4.00038		$(mm)^2$			kPa
LPM	6.6673E-02		$(mm)^2$			kPa
LPS	1.1112E-03		$(mm)^2$			kPa
m^3/D	9.6012E-02		$(mm)^2$			kPa
m^3/H	4.0005E-03		$(mm)^2$			kPa
m^3/M	6.6675E-05		$(mm)^2$			kPa
m^3/s	1.1112E-06		$(mm)^2$			kPa

Table 2-2. Liquid – Mass Rate of Flow

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Thermal Expansion Factor (Table A-11)	Flowing Specific Gravity	Differential Pressure
	Annubar Flow Constant C'					
W =	F_{na} ·	K ·	D^2 ·	F_{aa} ·	$\sqrt{\rho_f}$ ·	$\sqrt{h_w}$
PPD	8614.56		$(in)^2$			inch H_2O at 68 °F
PPH	358.94		$(in)^2$			inch H_2O at 68 °F
PPM	5.9823		$(in)^2$			inch H_2O at 68 °F
PPS	0.0997		$(in)^2$			inch H_2O at 68 °F
T(met)/hr	1.2645E-04		$(mm)^2$			kPa
kg/D	3.03471		$(mm)^2$			kPa
Kg/H	0.12645		$(mm)^2$			kPa
kg/M	2.1074E-03		$(mm)^2$			kPa
kg/S	3.5124E-05		$(mm)^2$			kPa

Table 2-3. Gas and Steam– Mass Rate of Flow

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Annubar Expansion Factor	Thermal Expansion Factor (Table A-11)	Flowing Specific Gravity	Differential Pressure
	Annubar Flow Constant C'						
W =	F_{na} ·	K ·	D^2 ·	Y_a ·	F_{aa} ·	$\sqrt{\rho_f}$ ·	$\sqrt{h_w}$
PPD	8614.56		$(in)^2$				inch H_2O at 68 °F
PPH	358.94		$(in)^2$				inch H_2O at 68 °F
PPM	5.9823		$(in)^2$				inch H_2O at 68 °F
PPS	0.0997		$(in)^2$				inch H_2O at 68 °F
T(met)/hr	1.2645E-04		$(mm)^2$				kPa
kg/D	3.03471		$(mm)^2$				kPa
Kg/H	0.12645		$(mm)^2$				kPa
kg/M	2.1074E-03		$(mm)^2$				kPa
kg/S	3.5124E-05		$(mm)^2$				kPa

Table 2-4. Volume Rate of Flow at STD Conditions - Gas

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Annubar Expansion Factor	Pressure Base Factor	Temperature Base Factor	Flowing Temperature Factor	Specific Gravity Factor	Supercomp Factor (Table 8)	Thermal Expansion Factor (Table 9)	Flowing Specific Gravity	Differential Pressure
Annubar Flow Constant C'												
Q_a =	F_{na} ·	K ·	D^2 ·	Y_a ·	F_{pb} ·	F_{tb} ·	F_{tf} ·	F_g ·	F_{pv} ·	F_{aa} ·	$\sqrt{\rho_f}$ ·	$\sqrt{h_w}$
SCFD	8116.1		$(in)^2$									H_20 @ 68° F
SCFH	338.17		$(in)^2$									H_20 @ 68° F
SCFM	5.6362		$(in)^2$									H_20 @ 68° F
NL/H	11.34700		$(mm)^2$									kPa
NL/M	0.18912		$(mm)^2$									kPa
NM^3/D	0.27234		$(mm)^2$									kPa
NM^3/H	1.1347E-02		$(mm)^2$									kPa
NM^3/M	1.8912E-04		$(mm)^2$									kPa
NM^3/S	3.1520E-06		$(mm)^2$									kPa

Table 2-5. Volume Rate of Flow at Act Conditions

Rate of Flow	Unit Conversion Factor	Annubar Flow Coefficient	Internal Pipe Diameter	Annubar Expansion Factor	Thermal Expansion Factor (Table 9)	Flowing Specific Gravity	Differential Pressure
	Annubar Flow Constant C'						
Q_a =	F_{na} ·	K ·	D^2 ·	Y_a ·	F_{aa} ·	$\sqrt{\frac{l}{\rho_f}}$ ·	$\sqrt{h_w}$
ACFD	8614.56		$(in)^2$				H_20 @ 68° F
ACFH	358.94		$(in)^2$				H_20 @ 68° F
ACFM	5.9823		$(in)^2$				H_20 @ 68° F
AL/H	126.4434		$(mm)^2$				kPa
AL/M	2.10739		$(mm)^2$				kPa
Am^3/D	3.03473		$(mm)^2$				kPa
Am^3/H	0.12645		$(mm)^2$				kPa
Am^3/M	2.1074E-03		$(mm)^2$				kPa
Am^3/S	3.5124E-05		$(mm)^2$				kPa

NOMENCLATURE

D — Internal diameter of pipe, inches (mm)

F_{aa} — Thermal Expansion Factor. This factor corrects for the flowing area change of the pipe at the Annubar location due to temperature effects. For 316 stainless steel Annubar primary elements mounted in carbon steel pipe, F_{aa} = 1.0000 for temperatures between 31 and 106 °F. See Table B-1 on page B-3 which includes thermal expansion factors for various pipe materials at several temperatures.

F_g — Specific Gravity Factor. This factor corrects the flow equation whenever the gas is not air. The factor can be calculated as:

$$F_g = \sqrt{\frac{1}{G}}$$

where, G = specific gravity of flowing gas, air = 1.000. For a more complete description of specific gravity, see “Density, Specific Weight, Specific Gravity” on page 1-4 and Appendix D: Related Calculations.

F_{na} — Units Conversion Factor. This factor is used to convert the flow rate to the desired set of units. Appendix D: Related Calculations shows an example of how the numerical value of F_{na} is derived from the hydraulic equation for a given set of input units.

F_{pb} — Pressure Base Factor. The Pressure Base Factors are calculated to give gas volumes at a pressure base of 14.73 psia (101.325 kPa abs). The pressure base factor can be calculated as:

$$F_{pb} = \frac{14.73}{\text{base pressure, psia}} \quad \text{OR} \quad F_{pb} = \frac{101.325}{\text{base pressure, kPa abs}}$$

F_{pv} — Supercompressibility Factor. The Supercompressibility Factor accounts for the deviation from the “ideal gas” laws. In the flow equations, gas volumes are assumed to vary with pressure and temperature in accordance with Boyle’s and Charles' laws (the “ideal gas” laws). Actually, the volume occupied by individual gases deviate, by a slight degree, from the volumes which the “ideal gas” laws indicate. The amount of deviation is a function of the composition of the gas and varies primarily with static pressure and temperature. The actual deviation may be obtained by a laboratory test conducted on a sample of the gas, carefully taken at line conditions of pressure and temperature.

The National Bureau of Standards, Circular 564, gives the compressibility factor (Z) of air and other pure gases. The relationship between supercompressibility factor and compressibility factor is as follows:

$$F_{pv} = \sqrt{\frac{1}{Z}}$$

Table A-9 on page A-12 gives an abbreviated listing of the supercompressibility factors for air. Practical relationships have been established by which this deviation can be calculated and tabulated for natural gases containing normal mixtures of hydrocarbon components, considering the presence of small quantities of carbon dioxide and nitrogen and also relating the deviation to the heating value of gas. The A.G.A. manual (NX-19), “Determination of Supercompressibility Factors for Natural Gas”, should be used for determination of F_{pv}.

F_{tb} — Temperature Base Factor. The Temperature Base Factors are calculated to give gas volumes at a base temperature of 60 °F (520°R) for English Units. In order to adapt the flow equation for use in SI units, the factor is calculated similarly at 16 °C (289.15 K). The factor can be calculated as:

$$F_{tb} = \frac{\text{temperature base (°F)} + 460}{520} \quad \text{OR} \quad F_{tb} = \frac{\text{temperature base (°C)} + 273.15}{288.15}$$

F_{tf} — Flowing Temperature Factor. The units conversion factor (F_{NA}) for volumetric flow of gases at standard conditions has been calculated assuming that the gas temperature flowing around the Annubar primary element is 60 °F (520 °R) or 16 °C (289 K). If measurement is made at any other flowing temperature, then the flowing temperature factor must be applied. The factor can be found in Table A-8 on page A-11 or calculated as:

$$F_{tf} = \sqrt{\frac{520}{\text{flowing temperature (°F)} + 460}} \quad \text{OR} \quad F_{tf} = \sqrt{\frac{288.15}{273.15 + \text{flowing temperature (°C)}}}$$

G — Specific Gravity of Flowing Liquid. Ratio of the density of the flowing fluid to the density of water at 60°F which is 63.3707 lbm/ft^3. See Table A-4 on page A-6 for specific gravities of various liquids.

h_w — Differential pressure produced by the Annubar primary element. For this handbook, the differential pressure is expressed as the height, in inches, of a water column at 68 °F at standard gravity (g_c = 32.174 ft/sec^2). In SI Units, the differential pressure is expressed in kPa.

h_w = inches H_2O at 68 °F (kPa)

K — Flow Coefficient. Equation 2-8 on page 2-9 defines the flow coefficient of an Annubar primary element. It is related to the diameter of the pipe and is generally expressed as a function of Reynolds Number. See “Reynolds Number” on page 1-8 for an explanation of Reynolds Number.

P_f Flowing Pressure. This is the static pressure, in absolute units, existing in the pipe. For this handbook, the pressures are expressed in psia (kPa abs).

Q_a Actual Volumetric Flow Rate. This term is the flow rate of the fluid passing the Annubar primary element in actual volume units per units of time. Examples are actual cubic feet per hour (ACFH), GPM, Am^3/h, etc.

Q_s Standard (Normal) Volumetric Flow Rate. This term is the flow rate of the fluid passing the Annubar primary element in standard volume units per unit of time. For some gases, especially fuel gases, the cubic foot is the unit of measurement. However, a cubic foot of gas has no absolute or comparative value unless the pressure and temperature of the gas are specified. A common unit used for evaluating rates of flow is standard cubic foot per hour (SCFH). This unit states how many cubic feet of gas per hour would be flowing around the Annubar primary element if the flowing pressure and temperature were equal to the base pressure and temperature. For this handbook, the base pressure is 14.73 psia (101.56 kPa abs) and the base temperature is 60 °F (520 °R) or 0 °C (273 K).

ρ_f Flowing Density. For this handbook, the densities are expressed in lbm/ft (kg/m^3). Appendix A: Fluid Properties and Pipe Data gives densities of various fluids.

Y_A Expansion Factor. When a gas flows around an Annubar primary element, the change in velocity is accompanied by a change in density. The expansion factor must be applied to correct for this change. The expansion factor also accounts for small changes in the internal energy of the molecules due to the temperature difference between the upstream and downstream pressure ports of the Annubar primary element. The variation of the expansion factor is small and the ratio of specific heats for commercial gases is sufficiently constant to warrant using a constant ratio of specific heat. Use the following algorithm to calculate the value of the gas expansion factor. This equation adjusts for density and internal energy effects of the gas as it flows around the Annubar primary element.

Equation 2-6. : Gas Expansion Factor

$$Y_a = 1 - (Y_1(1-B)^2 - Y_2)\frac{h_w}{P_f \Upsilon}$$

where:

Equation 2-7. : Blockage Equation

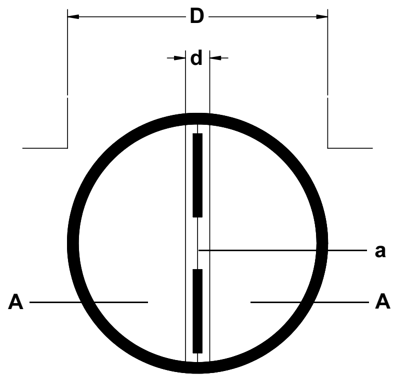
$B = \frac{4d}{\pi D}$ = Blockage
D = Internal Pipe Diameter in inches (cm)
d = See Table 2-7 on page 2-10
h_w = Differential pressure in inches (mm) of water column
P_f = Flowing line pressure in psia (kPa abs)
γ = Ratio of specific heats
Y_1= 0.011332 in English Units (0.31424 SI Units)
Y_2= 0.00342 in English Units (0.09484 SI Units)

Examples of gases with a specific heat ratio of 1.4 are: air, CO, H_2, NO, N_2 and O_2. Examples of gases with a specific heat ratio of 1.3 are: natural gas, ammonia, CO_2, Cl_2, H_2S, N_2O, SO_2, and steam.

Y_a is needed in all gas flow equations and requires the differential pressure be calculated first. If the differential pressure is not known, Y_a is assumed to be 1.000 and the differential pressure is calculated. Iteration is then necessary to determine a final value.

W Mass Rate of Flow. This term is the flow rate of the fluid passing the Annubar primary element in mass units per unit time.

Figure 2-1. Typical Cross Section



a = Annubar projected area = $d \cdot D$

A = Pipe inside area = $\frac{\pi D^2}{4}$

$B = \frac{a}{A} = \frac{4d}{\pi D}$

Flow Coefficient Reynolds Number Dependency

When the 485 Annubar primary element is used within the acceptable Reynolds Number range defined by Rosemount in Table 2-7 on page 2-10, the Annubar Primary element's flow coefficient will be independent of changing Reynolds Number. Any variations in the K-value with changing Reynolds Number are due to scatter and fall within ±0.75% of the published K-value.

A 485 Annubar primary element's K-factor independence of Reynolds number allows the user to measure a large range of Reynolds Numbers without need of a correction factor for changing Reynolds Numbers. The 485 Annubar primary element K-factor independence can be attributed to a constant separation point along the edges of its T-shaped sensor and the probe's ability to take a proper average of its sensing slots.

Flow Coefficient Theory

Rosemount was the first company to identify and utilize the theoretical equations linking self-averaging pitot tube flow coefficients to pipe blockage. This K-to-Blockage theoretical link establishes a higher degree of confidence in 485 Annubar K-factors than in flow meters that use only an empirical data base for determining their flow coefficients.

Signal

The signal generated by an Annubar can be divided into two major parts:

- the differential pressure contribution due to the Annubar 's shape (H_S)
- the differential pressure contribution due to the Annubar's blockage in the pipe (H_b).

Shape Differential

An Annubar primary element placed in an infinitely large pipe (with no confining walls) will still produce a differential pressure. This differential pressure is nearly twice that of a standard pitot tube, and is the result of a reduced low pressure on the downstream side. The upstream, or high pressure is caused by the fluid impacting the front of the Annubar primary element and is known as the stagnation pressure. The downstream, or low pressure is caused by the fluid traveling past the Annubar primary element, creating suction on the rear side. This suction phenomenon can be attributed to boundary layer flow separation.

Blockage Differential

An Annubar primary element is an obstruction in the pipe and therefore, reduces the cross-sectional area through which the fluid can pass. This reduced area causes the fluid to accelerate and hence, reduces its pressure. Therefore, the downstream pressure measurement of an Annubar primary element will be affected by the Annubar's blockage in the pipe.

Since an Annubar primary element uses the internal diameter of the pipe it is being inserted into as a throat diameter in its calculation of a flow rate, the Annubar primary element K-factor must compensate for the amount of obstructed area the sensor itself causes in the pipe. This is analogous to the velocity of approach factor for an orifice plate or a venturi meter.

By writing a mass balance and an energy balance around the Annubar primary element, and by dividing the differential pressure produced by the Annubar primary element into H_s and H_b, one can derive the relationship between the Annubar primary element K-factor and the Annubar primary element's blockage in the pipe. The derivation involves partial differential pressure components, and the integration of a K-blockage equation. The result is the following K vs. Blockage equation:

Equation 2-8. : K vs. Blockage

$$K_A = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$$

The constants C_1 and C_2 must be determined experimentally. Once C_1 and C_2 are determined, the equation above becomes the theoretical link between the Annubar primary element K-factor (K) and the Annubar primary element's blockage in the pipe (B). The values for constants C_1 and C_2 are shown in the table below:

Table 2-6. 485 Sensor Constants

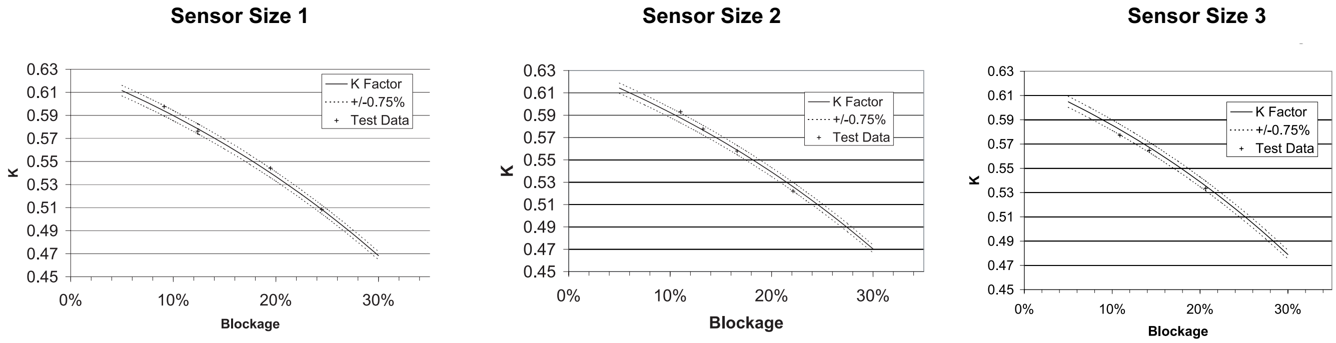
Coefficient	Sensor Size 1	Sensor Size 2	Sensor Size 3
C_1	– 1.515	– 1.492	– 1.5856
C_2	1.4229	1.4179	1.3318

The Importance of the Flow Coefficient, or K vs. B Theory

As with any other meter, the 485 Annubar primary element's accuracy is only as good as its flow coefficient (K-factor). Rosemount has tested a representative sample of flowmeters and empirically determined flow coefficients. For Annubars, these flow coefficients are plotted against the meter's blockage. Curve fitting techniques combined with flow coefficient theory are applied to the base line data to generate equations that predict flow coefficients in untested line sizes and untested Reynolds Number ranges. Please see the 485 Annubar Flow Test Data Book (document number 00821-0100-4809, Rev AA) for a more detailed discussion of this topic.

Provided the theory is based on the proper physics, these relationships are immune to minor variation in test data. Using a theoretical basis (in addition to empirical testing) for the prediction of untested flow coefficients provides a much higher degree of confidence in the untested values. The graphs in Figure 2-2 show that empirical data agree with a plot of the K vs. Blockage Equation.

Figure 2-2. K vs. BLOCKAGE



Operating Limitations

For an Annubar primary element to operate accurately, the flowing fluid must separate from the probe at the same location (along the edges of the T-shape sensor). Drag coefficients, lift coefficients, separation points, and pressure distributions around bluff bodies are best compared by calculating the “rod” Reynolds Number. There is a minimum rod Reynolds Number at which the flowing fluid will not properly separate from the edges of a T-shape sensor. The minimum rod Reynolds Numbers for the Rosemount 485 are:

Table 2-7. Reynolds Number and Probe Width

Sensor Size	Probe Width (d)	Minimum Reynolds Number
1	0.590-in. (1.4986 cm)	6000
2	1.060-in (2.6924 cm)	12500
3	1.935-in (4.915 cm)	25000

Above these rod Reynolds Numbers 485 Annubar primary elements will operate accurately.

To determine the rod Reynolds Number at any given flowrate, use the following relationship:

$$Re_{rod} = \frac{dV\rho}{12\mu} \qquad \text{OR} \qquad Re_{rod} = \frac{dV\rho}{100\mu}$$

where,
ρ = fluid density in lbm/ft^3 (kg/m^3)
d = probe width in inches (cm)
V = velocity of fluid in feet per second (m/s)
μ = fluid viscosity in lbm/ft-sec (kg/m-s)

When determining the minimum operating flow rate for an Annubar primary element, one should also consider the capability of the secondary instrumentation (differential pressure transmitters, manometers, etc.).

The upper operating limit for 485 Annubar primary elements is reached when any one of the following criteria is met:

1. The fluid velocity reaches the structural limit of the Annubar.
2. The fluid velocity reaches a choked flow condition at the Annubar (gas).
3. Cavitation occurs on the downstream side of the Annubar.

Flow Calculation Examples:

Problem:

Oil with a specific gravity of 0.825 is flowing at a rate of 6000 GPM. The 20-in. standard wall (ID = 19.26-in.) carbon steel pipeline has a pressure of 75 psig and a temperature of 100°F. What is the differential pressure (h_w) that a Sensor Size 2 485 Annubar primary element would measure?

Solution:

$$h_w = \left(\frac{Q_a}{C'}\right)^2$$ *(from Equation 2-1 on page 2-1)*

Qa = 600 GPM

$$C' = F_{na} \cdot K \cdot D^2 \cdot F_{aa} \cdot \sqrt{\frac{1}{G_f}}$$ *(from Equation 2-1 on page 2-1)*

where:

F_{na} = 5.6664

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1(1 - C_2 B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.060)}{19.25\pi} = 0.0701$$ *(from Equation 2-7 on page 2-7)*

$$C_1 = -1.492$$ *(from Table 2-6 on page 2-9)*

$$C_2 = 1.4179$$ *(from Table 2-6 on page 2-9,*

so:

$$K = \frac{(1 - 1.4179 \times 0.0701)}{\sqrt{1 - (-1.492) \times (1 - 1.4179 \times 0.0701)^2}} = 0.6058$$

$$D^2 = 19.26^2 = 370.9476$$

F_{aa} = 1.000

$$\sqrt{\frac{1}{G_f}} = \sqrt{\frac{1}{0.825}} = 1.101$$

so:

$$C = 5.6664 \cdot 0.6058 \cdot 370.9476 \cdot 1.000 \cdot 1.101 = 1401.9625$$

and:

$$h_w = \left(\frac{6000}{1401.9625}\right)^2 = 18.316 \qquad \text{inch}H_2O$$

Problem:

Oil with a specific gravity of 0.825 is flowing at a rate of 22,700 LPM. The 50 cm inside diameter carbon steel pipeline has a pressure of 517 kPa and a temperature of 38 °C. What is the differential pressure (h_w) that a Sensor Size 2 485 Annubar primary element would measure?

Solution:

$$h_w = \left(\frac{Q_a}{C'}\right)^2$$ *(from Equation 2-1 on page 2-1)*

Qa = 22700 LPM

(from Table 2-2 on page 2-4)

$$C' = F_{na} \cdot K \cdot D^2 \cdot F_{aa} \cdot \sqrt{\frac{1}{G_f}}$$ *(from Equation 2-1 on page 2-1)*

where:

F_{na} = 0.066673

$$K = \frac{(1 - C_2B)}{\sqrt{1 - C_1(1 - C_2B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(2.6924)}{50\pi} = 0.0686$$ *(from Table 2-6 on page 2-9)*

(from Table 2-6 on page 2-9)

$$C_1 = -1.492$$

(from Table 2-6 on page 2-9)

$$C_2 = 1.4179$$

so:

$$K = \frac{(1 - (1.4179 \cdot 0.0686))}{\sqrt{1 - ((-1.492) \cdot (1 - (1.4179 \cdot 0.0686))^2)}} = 0.6065$$

$$D^2 = 500^2 = 250000$$

F_{aa} = 1.000

$$\sqrt{\frac{1}{G_f}} = \sqrt{\frac{1}{0.825}} = 1.101$$

so:

$$C' = 0.066673 \cdot 0.6065 \cdot 250000 \cdot 1.000 \cdot 1.101 = 11130.33$$

and:

$$h_w = \left(\frac{22700}{11130.33}\right)^2 = 4.159 \quad \text{kPa}$$

Problem:

Steam at 500 psia and 620 °F is flowing in a 24-in. ID carbon steel pipe. The measured differential pressure on a Sensor Size 3 485 Annubar primary element is 15-in H_2O. What is the flowrate in PPH?

Solution:

$$W = C' \cdot \sqrt{h_w}$$ *(from Equation 2-2 on page 2-1)*

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{aa} \cdot \sqrt{\rho_f}$$ *(from Equation 2-3 on page 2-1)*

where:

$F_{na} = 358.94$ *(from Table 2-2 on page 2-4)*

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.920)}{24\pi} = 0.1019$$ *(from Equation 2-7 on page 2-7)*

$C_1 = (-1.5856)$ *(from Table 2-6 on page 2-9)*

$C_2 = 1.3318$ *(from Table 2-6 on page 2-9)*

so:

$$K = \frac{(1 - (1.3318 \cdot 0.1019))}{\sqrt{1 - ((-1.5856) \cdot (1 - (1.3318 \cdot 0.1019))^2)}} = 0.5848$$

$$D^2 = 24^2 = 576$$

$$Y_a = 1 - (0.011332(1 - B)^2 - 0.00342)\frac{h_w}{P_f \Upsilon}$$ *(from Equation 2-6 on page 2-7)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.920)}{24\pi} = 0.1019$$ *(from Equation 2-7 on page 2-7)*

$H_w = 15inH_2O$
$P_f = 500psia$
$\Upsilon = 1.3$

so:

$$Y_a = 1 - (0.011332(1 - 0.1019)^2 - 0.00342)\frac{15}{500 \cdot 1.3} = 0.9999$$

$F_{aa} = 1.008$

ρ_f *per ASME steam tables*

$$\sqrt{\rho_f} = \sqrt{0.8413} = 0.9172$$

so

$$C' = 358.94 \cdot 0.5848 \cdot 576 \cdot 0.9999 \cdot 1.008 \cdot 0.9172 = 111771.96$$

$$W = 111771.96 \cdot \sqrt{15} = 432890.93 \quad PPH$$

Problem:

Steam at 3500 kPa abs and 350 °C is flowing in a 60.96 cm ID carbon steel pipe. The measured differential pressure on a Sensor Size 3 485 Annubar primary element is 7.5 kPa. What is the flowrate in kg/hr?

Solution:)

$$W = C' \cdot \sqrt{h_w}$$ *(from Equation 2-2 on page 2-1)*

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{aa} \cdot \sqrt{\rho_f}$$ *(from Equation 2-3 on page 2-1)*

where:

$F_{na} = 0.12645$

$$K = \frac{(1 - C_2B)}{\sqrt{1 - C_1(1 - C_2B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(4.9149)}{60.96\pi} = 0.1027$$ *(from Equation 2-7 on page 2-7)* *(from Table 2-6 on page 2-9)*

(from Table 2-6 on page 2-9)

$$C_1 = -1.5856$$

(from Table 2-6 on page 2-9)

$$C_2 = 1.3318$$

so:

$$K = \frac{(1 - 1.3318 \times 0.1027)}{\sqrt{1 - ((-1.5856) \cdot (1 - (1.3318 \cdot 0.1027))^2)}} = 0.5845$$

$$D^2 = 609.6^2 = 371612.16$$

$$Y_a = 1 - (0.31424(1 - B)^2 - 0.09484)\frac{h_w}{P_f\Upsilon}$$ *(from Equation 2-6 on page 2-7)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(4.9149)}{60.96\pi} = 0.1027$$ *(from Equation 2-7 on page 2-7)*

$H_w = 7.5kPa$

$P_f = 3500kPaA$

$\Upsilon = 1.3$

so:

$$Y_a = 1 - (0.31424(1 - 0.1027)^2 - 0.09484)\frac{7.5}{3500 \times 1.3} = 0.9997$$

(from Table B-1 on page B-1)

$$F_{aa} = 1.009$$

$$\sqrt{\rho_f} = \sqrt{13.0249} = 3.609$$ ρ_f *per ASME steam tables*

so

$$C' = 0.12645 \cdot 0.5845 \cdot 371612.16 \cdot 0.9997 \cdot 1.009 \cdot 3.609 = 99986.42$$

$$W = 99986.42 \cdot \sqrt{7.5} = 273824.1\,kg/h$$

Problem:

Natural gas with a specific gravity of 0.63 is flowing in a 12-in. schedule 80 carbon steel pipe. the operating pressure is 1264 psia. The operating temperature is 120 °F. For a Sensor Size 2 485 Annubar primary element, determine the differential pressure (h_w) for a flowrate of 6 MM SCFH at a base temperature of 60 °F and a pressure of 14.73 psia.

Solution:

$$h_w = \frac{1}{P_f} \cdot \left(\frac{Q_s}{C'}\right)^2$$

$$Q_s = 6000000\,SCFH$$

$$P_f = 1264\,psia$$

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{pb} \cdot F_{tb} \cdot F_{tf} \cdot F_g \cdot F_{pv} \cdot F_{aa}$$ *(from Equation 2-4 on page 2-1)*

where:

$F_{na} = 338.17$

$$K = \frac{(1 - C_2B)}{\sqrt{1 - C_1(1 - C_2B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.060)}{11.37\pi} = 0.1186$$ *(from Equation 2-7 on page 2-7)*

$C_1 = -1.492$ *(from Table 2-4 on page 2-5)*

$C_2 = 1.4179$ *(from Table 2-4 on page 2-5)*

so:

$$K = \frac{(1 - (1.4179 \cdot 0.1186))}{\sqrt{1 - ((-1.492) \cdot (1 - (1.4179 \cdot 0.1186))^2)}} = 0.5835$$

$$D^2 = 11.376^2 = 129.41$$

The differential pressure h_w is required to calculate Y_a. Since h_w is not known, assume Y_a = 1 and verify the results

$$F_{pb} = \frac{14.73}{\text{base pressure, psia}} = \frac{14.73}{14.73} = 1$$

$$F_{tb} = \frac{\text{temperature base (°F) + 460}}{520} = \frac{60 + 460}{520} = 1$$

$$F_{tf} = \sqrt{\frac{520}{\text{flowing temperature (°F) + 460}}} = \sqrt{\frac{520}{120 + 460}} = 0.9469$$

$$F_g = \sqrt{\frac{1}{G}} = \sqrt{\frac{1}{0.63}} = 1.2599$$

$$F_{pv} = \sqrt{\frac{1}{Z}} = \sqrt{\frac{1}{0.8838}} = 1.0637$$

(Compressibility factor for natural gas from A.G.A Report No. 3)

$$F_{aa} = 1.001$$

so:

$$C = 338.17 \times 1.5835 \times 129.41 \times 1 \times 1 \times 1 \times 0.9469 \times 1.2599 \times 1.0637 \times 1.001 = 32436.74$$

$$h_w = \frac{1}{P_f} \cdot \left(\frac{Q_s}{C^I}\right)^2 = \frac{1}{1264} \cdot \left(\frac{6000000}{32436.74}\right)^2 = 27.07 \text{inH}_2\text{O}$$

Now the value of Y_a, assumed above, can be checked:

$$Y_a = 1 - (0.011332(1 - B)^2 - 0.00342)\frac{h_w}{P_f \Upsilon}$$

(from Equation 2-6 on page 2-7)

where:

$$B = \frac{4d}{\pi D} = \frac{4(1.060)}{11.37\pi} = 0.1186$$

(from Equation 2-7 on page 2-7)

$$H_w = 27.07 \text{inchH}_2\text{O}$$

$$P_f = 1264 \text{psia}$$

$$\Upsilon = 1.3$$

so:

$$Y_a = 1 - (0.011332(1 - 1186)^2 - 0.00342)\frac{27.07}{1264 \times 1.3} = 1$$

The assumed and calculated value are the same. therefore, the value of h_w = 24.27 inch H_2O is the correct answer.

Problem:

Natural gas with a specific gravity of 0.63 is flowing in a 300 mm ID carbon steel pipe. The operating pressure is 8700 kPA abs and the operating temperature is 50 °C. For a Sensor Size 2 485 Annubar primary element, determine the differential pressure (h_w) for a flowrate of 1700 Nm^3/m at a base temperature of 0 °C and a pressure of 101.325 kPa.

Solution:

$$h_w = \frac{1}{P_f} \cdot \left(\frac{Q_s}{C'}\right)^2$$ *(from Equation 2-4 on page 2-1)*

$$Q_s = 1700 Nm^3/m$$

$$P_f = 8700 kPa$$

$$C' = F_{na} \cdot K \cdot D^2 \cdot Y_a \cdot F_{pb} \cdot F_{tb} \cdot F_{tf} \cdot F_g \cdot F_{pv} \cdot F_{aa}$$ *(from Equation 2-4 on page 2-1)*

where:

$F_{na} = 0.00018912$

$$K = \frac{(1 - C_2 B)}{\sqrt{1 - C_1 (1 - C_2 B)^2}}$$ *(from Equation 2-8 on page 2-9)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(26.924)}{300\pi} = 0.1143$$ *(from Equation 2-7 on page 2-7)*

$$C_1 = -1.492$$ *(from Table 2-6 on page 2-9)*

$$C_2 = 1.4179$$ *(from Table 2-6 on page 2-9)*

so:

$$K = \frac{1 - (1.4179 \cdot 0.1143)}{\sqrt{1 - ((-1.492) \cdot (1 - (1.4179 \cdot 0.1143))^2)}} = 0.5856$$

$$D^2 = 300^2 = 90000$$

The differential pressure h_w is required to calculate Y_a. Since h_w is not known, assume $Y_a = 1$ and verify the results.

$$F_{pb} = \frac{101.325}{\text{base pressure, kPa abs}} = \frac{101.325}{101.325} = 1$$

$$F_{pb} = \frac{\text{temperature base (°C)} + 273.15}{288.15} = \frac{0 + 273.15}{288.15} = 0.9479$$

$$F_{tf} = \sqrt{\frac{288.15}{273.15 + \text{flowing temperature (°C)}}} = \sqrt{\frac{288.15}{273.15 + 50}} = 0.9443$$

$$F_g = \sqrt{\frac{1}{G}} = \sqrt{\frac{1}{0.63}} = 1.2599$$

$$F_{pv} = \sqrt{\frac{1}{Z}} = \sqrt{\frac{1}{0.876}} = 1.0684$$

$$F_{aa} = 1.001$$

so:

$$C' = 0.00018912 \cdot 0.5856 \cdot 90000 \cdot 1 \cdot 0.9479 \cdot 1 \cdot 0.9443 \cdot 1.2599 \cdot 1.0684 \cdot 1.001 = 12.0215$$

$$h_w = \frac{1}{P_f} \cdot \left(\frac{Q_s}{C'}\right)^2 = \frac{1}{8700} \cdot \left(\frac{1700}{12.0215}\right)^2 = 2.2986\text{kPa}$$

Now the value of Y_a, assumed above, can be checked:

$$Y_a = 1 - (Y_1(1-B)^2 - Y_2)\frac{h_w}{P_f \Upsilon}$$ *(from Equation 2-6 on page 2-7)*

where:

$$B = \frac{4d}{\pi D} = \frac{4(26.924)}{300\pi} = 0.1143$$ *(from Equation 2-7 on page 2-7)*

$$H_w = 2.2986\text{kPa}$$

$$P_f = 8700\text{kPa}$$

$$\Upsilon = 1.3$$

so:

$$Y_a = 1 - (0.31424 \cdot (1 - 0.1143)^2 - 0.09484)\frac{2.2986}{8700 \cdot 1.3} = 1$$

The assumed and calculated value are the same. therefore, the value of h_w = 2.2986kPa is the correct answer.